Simulation of Servo Loops in Atomic Clock Ensemble in Space (ACES)

J. K. Dam[1], W. Schäfer[1], G. Hejc[1], M. P. Hess[2], L. Stringhetti[2], J. Kehrer[2], L. Cacciapuoti[3]

[1]TimeTech GmbH, Curiestr. 2, 70563 Stuttgart, Germany
Email: joydeep.dam@timetech.de

[2]EADS Astrium, 88093 Friedrichshafen, Germany
Email: Marc-Peter.Hess@astrium.eads.net

[3]European Space Agency, ESTEC, 2200 AG Noordwijk, Netherlands
Email: Luigi.Cacciapuoti@esa.int

*Abstract***— Atomic Clock Ensemble in Space (ACES) is an ESA mission that will operate a new generation of atomic clocks in micro-gravity environment of the International Space Station (ISS). Two high performance atomic clocks in ACES: the Space Hydrogen Maser (SHM) and PHARAO, a clock based on laser cooled cesium atoms, are compared on-board the ISS and locked together by two separate servo-loops: short and long term servo loop. This paper presents a simulation model for the complete system that evaluates the performance of the ACES clock signal at its output using frequency domain techniques. The results of ACES servo loop simulations for both PHARAO ground and space models are presented. Result for PHARAO ground model is compared to ACES system level test that has been performed in Toulouse, 2009.**

1 INTRODUCTION

Atomic Clocks Ensemble in Space (ACES) is an ESA fundamental physics mission based on the operation of atomic clocks with high stability and accuracy in the microgravity environment of the International Space Station (ISS). ACES will explore and demonstrate high performance atomic clocks in space environment and distribute a stable and accurate frequency reference that will be used for space-to-ground and ground-to-ground clock comparisons. These comparisons will allow accurate tests of Einstein's theory of general relativity.

Major elements of ACES are two high-performance atomic clocks: the Space Hydrogen Maser (SHM) and PHARAO, a clock which is based on laser cooling cesium (CS) atoms. The two clocks are compared on-board the ISS and locked together by two separate servo-loops that have two-fold function: to avoid low frequency beatings and optimize the frequency stability and accuracy of the ACES clock signal. Short Term Servo Loop (STSL) is a phase-locked loop (PLL) of time constant 1s to 10s, stabilizing local oscillator of PHARAO (SH – Source Hyperfréquence) on SHM signal. Long Term Servo Loop (LTSL) is a frequency-locked loop (FLL) of time constant of ~3000sec (~1day when the system is operated on ground), stabilizing SHM on the error signal generated by PHARAO on cesium atoms. STSL provides ACES clock signal with short- and midterm frequency stability of SHM, while LTSL provides ACES clock signal with the long-term stability and accuracy of PHARAO.

2 SIMULATION MODEL

The simulation model includes complete ACES servo-loop system and evaluates the performance of the ACES clock signal at its output using frequency domain techniques with the assumption that all the noise sources are uncorrelated. In comparison to time domain approaches, frequency domain techniques take less computational resources to calculate Allan Deviation at long integration times to the desired confidence level. All the noise sources can be reliably and easily simulated using the appropriate Power-Law. For a linear system, the output power spectral density (PSD) can be obtained from the transfer functions of each element and from the power spectral densities of the various independent noise sources. The resulting power spectral density is finally used to compute the Allan deviation at the system output. The contribution of each servo-loop to the Allan deviation at the system output due to independent noise sources is evaluated and used for design trade-off and troubleshooting. Following block diagram shows the entire ACES servo-loop system with noise sources. This model has been used to simulate the result of the measurements performed during the ACES EM test campaign.

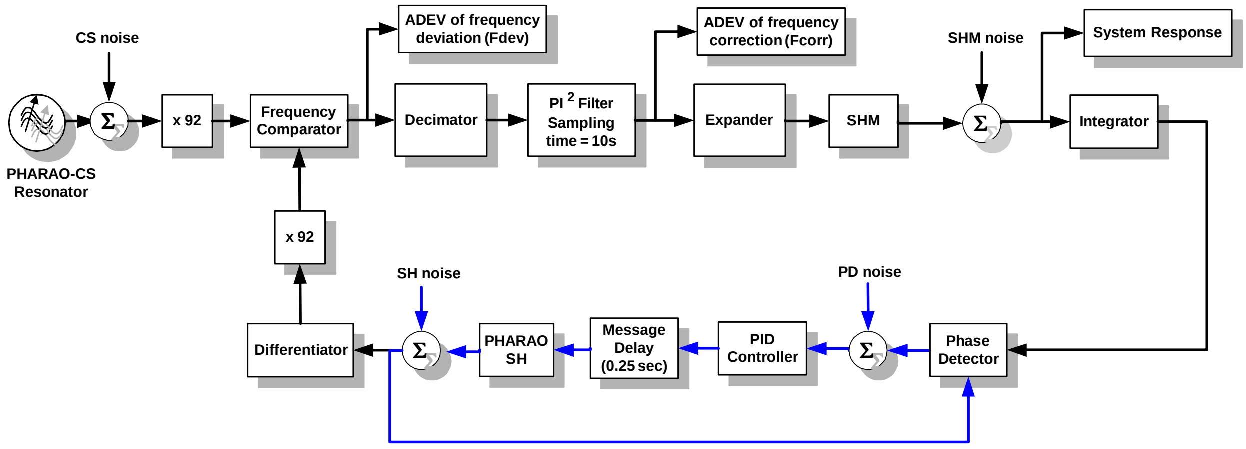

Fig. 1: Simulation model for complete ACES servo-loop system with noise sources, where blue arrows show STSL and black arrows show LTSL

The system provides an output frequency from PHARAO-CS resonator at 100MHz. Both SHM and SH also generate 100MHz signals. During STSL (showing in blue arrows), phase comparison between 100MHz signals from SHM (an integrator converts SHM 100MHz signal into phase) and SH is performed in the phase detector (PD). Based on the phase error computed by PD, STSL adjust the phase of SH in such a way that it is phase-locked to reference SHM signal.

During LTSL, phase corrected SH 100MHz signal will be converted from phase to frequency using a differentiator and after that both reference CS and SH 100MHz signals are up-converted to 9.2 GHz. They will be compared in a frequency comparator (FC) and the frequency comparison value will be corrected by a double integral low pass filter, denoted as PI^2 filter with sampling time of 10sec. In the simulation model, to avoid aliasing effect due to down-sampling from 0.25sec to 10sec and up-sampling from 10sec to 0.25sec again, a decimator and an expander (both 1st order digital low pass filters with sampling time 0.25sec), respectively before and after PI^2 filter, are inserted. Finally the frequency correction value is down-converted by a factor of 92 and is given as input to the SHM 100MHz signal to adjust the frequency of SHM 100MHz signal and therefore SHM is frequency-locked to CS resonator.

3 NOISE MODEL

3.1 PHARAO-CS Resonator Noise Model

The noise in PHARAO CS resonator is identified as the white frequency noise. The relative frequency noise PSD of the CS resonator for ground, $S_{gCS}(f)$ and space model, $S_{sCS}(f)$ therefore can be written respectively as follows:

$$S_{gCS}(f) = \left(\frac{2.6e-9}{f_0^{\;2}} \right) \tag{1}$$

$$S_{sCS}(f) = \left(\frac{4.0e-9}{f_0^{\;2}} \right) \tag{2}$$

3.2 SHM Noise Model

The noise contributions of SHM are white phase, white frequency and flicker frequency noise. If the noises are independent to each other, the relative frequency noise PSD for SHM, $S_{SHM}(f)$ can be written as:

$$S_{SHM}(f)=\left(\frac{9.0e-10\times f^{2}}{f_{0}^{2}}\right)+\left(\frac{15e-12}{f_{0}^{2}}\right)+\left(\frac{22e-10}{f_{0}^{2}\times f}\right) \tag{3}$$

3.3 PHARAO-SH Noise Model

Noise contributions of PHARAO-SH are random walk, flicker frequency and white phase noise. When the noises are uncorrelated to each other, the relative frequency noise PSD for SH, $S_{SH}(f)$ can be written as:

$$S_{SH}(f)=\left(\frac{4.58e-14}{f_{0}^{2}\times f^{2}}\right)+\left(\frac{2.34e-11}{f_{0}^{2}\times f}\right)+\left(\frac{2.1e-10\times f^{2}}{f_{0}^{2}}\right) \tag{4}$$

3.4 Phase Detector Noise Model

The noise in Phase Detector (PD) is defined as the white phase noise. The relative frequency noise PSD of the PD, $S_{PD}(f)$ can be written as follows:

$$S_{PD}(f)=\left(\frac{1.2e-10\times f^{2}}{f_{0}^{2}}\right) \tag{5}$$

4 SYSTEM TRANSFER FUNCTIONS

From the system model shown in Fig.1, equivalent transfer function equations can be built using Z transform and Fourier frequency domains. Simulation sampling time, T_S is 0.25 sec. Conversion between Z transform and Fourier frequency domain is done using the equation:

$$z=e^{j2\pi f T_S} \tag{6}$$

The transfer functions relating LTSL output frequency (F_{OUT}) to input noise sources of CS, SHM, PD and SH are denoted in Fourier frequency domain respectively as $H_{CS}(f)$, $H_{SHM}(f)$, $H_{PD}(f)$ and $H_{SH}(f)$ and they are as follows:

$$H_{CS}(f)=\frac{F_{OUT}(f)}{CS(f)} \tag{7}$$

$$H_{SHM}(f)=\frac{F_{OUT}(f)}{SHM(f)} \tag{8}$$

$$H_{PD}(f)=\frac{F_{OUT}(f)}{PD(f)} \tag{9}$$

$$H_{SH}(f)=\frac{F_{OUT}(f)}{SH(f)} \tag{10}$$

4.1 Proportional-Integral-Differentiation Controller Transfer Function

In STSL, the phase difference value is fed to the proportional-integral-differentiation (PID) controller, which finally provides the feedback phase difference value to PHARAO MWS. As mentioned earlier, STSL is a PLL. According to the simulation model presented here, we have no extra filter inside the PLL, but the PID controller inside STSL is acting itself as a low pass filter.

In z-domain, transfer function of the PID controller with proportional gain K_P, integral gain K_I and differentiation gain K_D, in STSL can be written as follows:

$$H_{PID}(z) = K_P + \frac{K_I}{(1-z^{-1})} + K_D(1-z^{-1}) \tag{11}$$

4.2 Proportional-Double Integral Filter Transfer Function

LTSL PI^2 controller has the transfer function in z domain as follows:

$$H_{PID}(z) = K_A + \frac{K_B}{(1-z^{-1})} + \frac{K_C}{(1-z^{-1})^2} \tag{12}$$

5 POWER SPECTRAL DENSITY OF THE SYSTEM OUTPUT FREQUENCY

Using equations 1, 2, 3 and 4 we compute the power spectral density of the system output individually for noisy CS ground (S_{gCSOUT}) and space model (S_{sCSOUT}), SHM (S_{SHMOUT}), PD (S_{PDOUT}) and SH (S_{SHOUT}) as:

$$S_{gCSOUT}(f) = |H_{CS}(f)|^2 \times S_{gCS}(f) \tag{13}$$

$$S_{sCSOUT}(f) = |H_{CS}(f)|^2 \times S_{sCS}(f) \tag{14}$$

$$S_{SHMOUT}(f) = |H_{SHM}(f)|^2 \times S_{SHM}(f) \tag{15}$$

$$S_{PDOUT}(f) = |H_{PD}(f)|^2 \times S_{PD}(f) \tag{16}$$

$$S_{SHOUT}(f) = |H_{SH}(f)|^2 \times S_{SH}(f) \tag{17}$$

6 COMPUTATION OF ALLAN DEVIATION AT THE SYSTEM OUTPUT FREQUENCY

Behavior of time or frequency system is normally characterized by the Allan Deviation (ADEV) or square root of the Allan Variance (AVAR, denoted as $\sigma_y^2(\tau)$, where τ is the frequency sample time). The relationship between AVAR and the relative frequency noise PSD, $S_y(f)$ for high frequency cut-off of the system, f_h can be written according to [1]:

$$\sigma_y^2(\tau) = 2\int_0^{fh} S_y(f) \frac{\sin^4(\pi f \tau)}{(\pi f \tau)^2} df \tag{18}$$

To calculate AVAR and therefore ADEV using above equation, we choose Simpson's Numerical Integration method due to complexity in carrying out integration analytically. We simulated a maximum τ of 40000 sec using 1E6 divisions in between low (0Hz) and high cut-off (1Hz) frequencies. If AVAR contributions of noisy CS, SHM, PD and SH at the system output are respectively, $\sigma_{CSOUT}^2(\tau)$, $\sigma_{SHMOUT}^2(\tau)$, $\sigma_{PDOUT}^2(\tau)$ and $\sigma_{SHOUT}^2(\tau)$, then, ACES system total AVAR ($\sigma_{TOT}^2(\tau)$,) is simply the summation of the AVAR of all individual noise sources at the system output and can be written as:

$$\sigma^2_{TOT}(\tau) = \sigma^2_{CSOUT}(\tau) + \sigma^2_{SHMOUT}(\tau) + \sigma^2_{PDOUT}(\tau) + \sigma^2_{SHOUT}(\tau) \quad (19)$$

7 RESULTS

Fig.2 shows system ADEV for both space and ground PHARAO-CS. It can be seen that till approximately 1000sec system response following SHM while after 4000sec the system ADEV following PHARAO-CS. From the occurrence of the bump at integration time approximately 4000sec, we can assume that the control loop has the time constant approximately 1 Hour with the choice of loop parameters mentioned on the top of the figure.

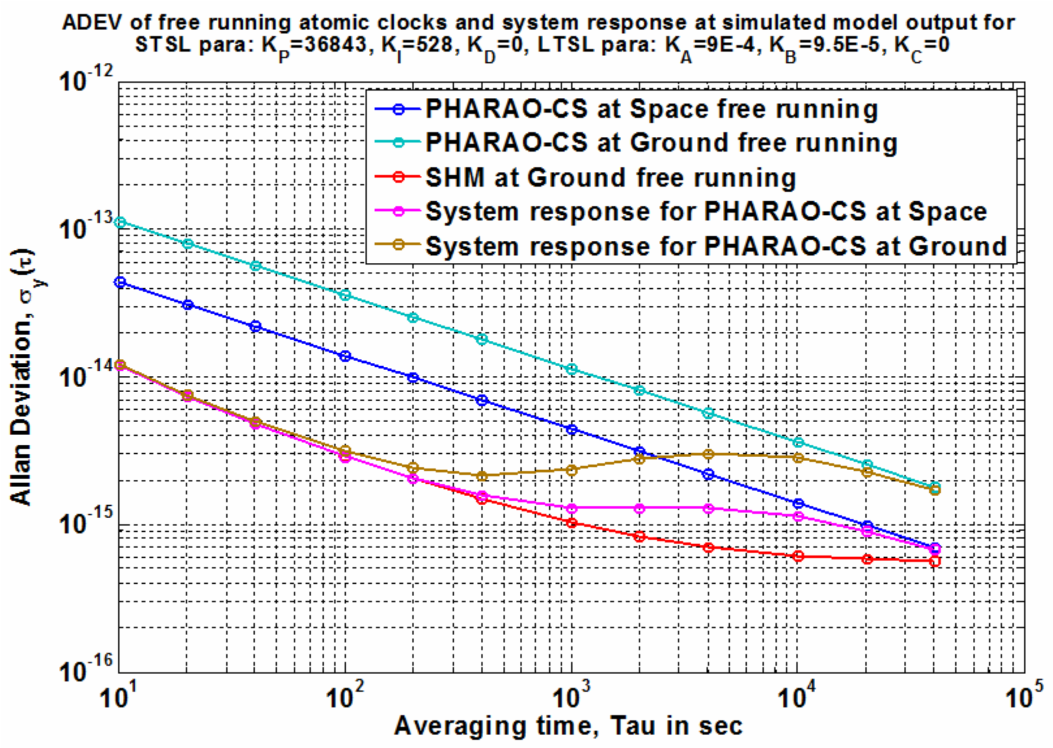

Fig. 2: ADEV of free running ACES atomic clocks and system response at simulated model output

Fig.3 shows the comparison between measured and simulated ADEVs for frequency deviation, f_{dev} (at 9.2GHz) and frequency correction, f_{corr} (at 100MHz). For integration times longer than the time constant of the LTSL, (approximately tau = 1000 sec), the ADEV of the frequency deviation has 1/Tau slope. For shorter integration times, the ADEV provides a measurement of PHARAO stability with respect to SHM.

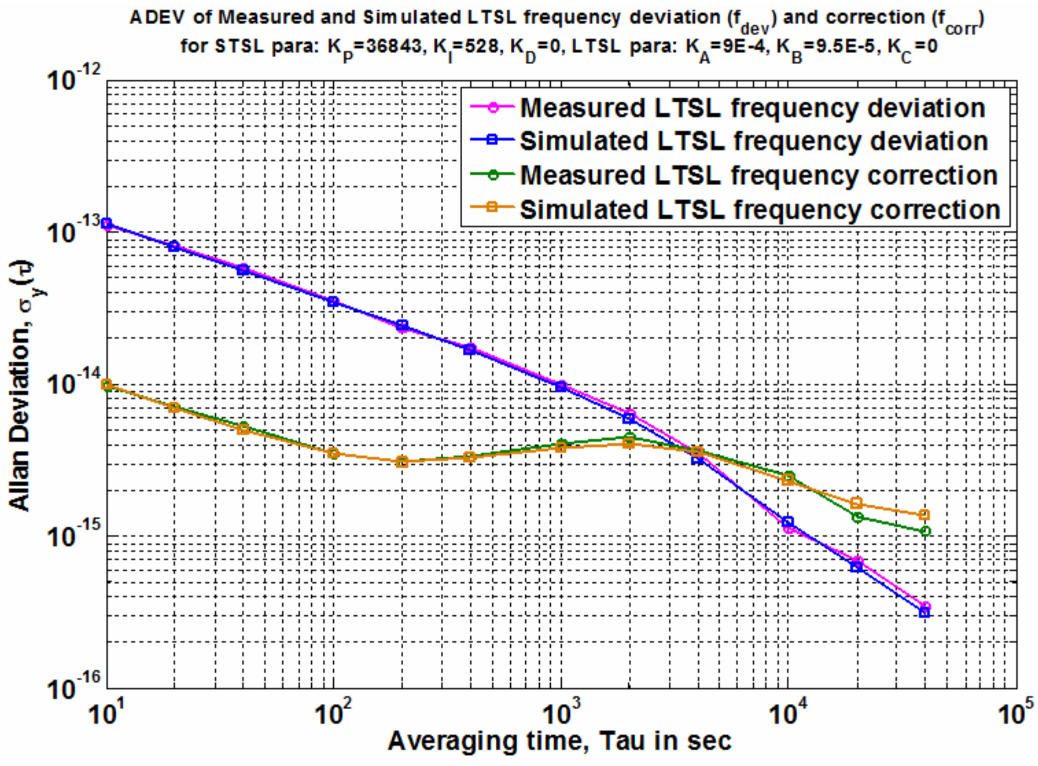

Fig. 3: ADEV comparison between measured and simulated frequency deviation and frequency correction

8 DISCUSSION

It is demonstrated that the simulated ADEVs are in agreement with the measured ADEVs, which proves the validity of the simulation model.

REFERENCES

[1] J. Barnes et. Al, "Characterization of Frequency Stability", IEEE Transactions on Instrumentation and Measurement, Vol. IM-20, No. 2, May 1971